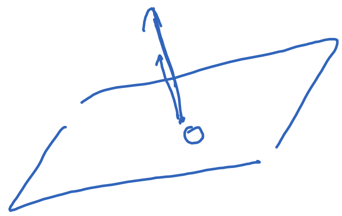
Lecture 26

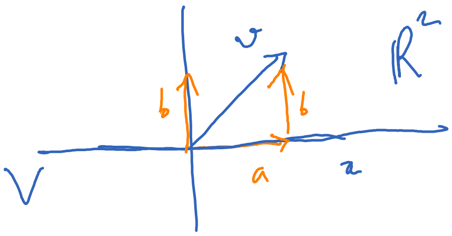
Thursday, December 3, 2020 3:18 PM

V subspace of $\mathbb{R}^n$.

$V^\perp = \{v \in \mathbb{R}^n : v \perp V\}$



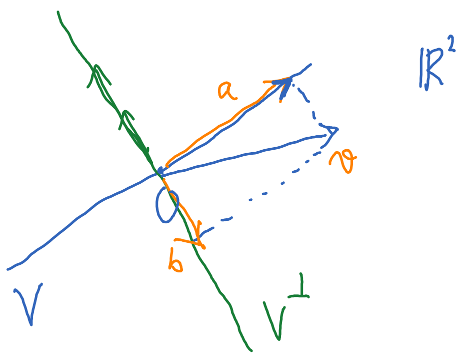
$v \in \mathbb{R}^n$



$v = \underline{a} + \underline{b}$ orthogonal decomposition

$a \in V$

$b \in V^\perp$



$a \in V$

$b \in V^\perp$

$$A = \begin{bmatrix} — & v_1 & — \\ — & v_2 & — \\ & \vdots & \\ — & v_m & — \end{bmatrix}$$

$A \ldots m \times n$

$\text{row} A = \text{span}\{v_1, v_2, \ldots, v_m\}$

subspace of $\mathbb{R}^n$

what is the $\underbrace{\text{orthogonal complement of row}(A)}_{(\text{row}(A))^\perp}$?

$$(\mathrm{row}(A))^{\perp} = \mathrm{null}(A)$$

Given a <u>subspace V of $\mathbb{R}^n$</u>, write $V = \mathrm{span}\{v_1, \dots, v_m\}$

$$A = \underbrace{\begin{bmatrix} - v_1 - \\ - v_2 - \\ \vdots \\ - v_m - \end{bmatrix}}$$

$V = \mathrm{row}(A)$

$V^{\perp} = \underline{\mathrm{null}(A)}$

$\dim V + \dim V^{\perp} = n$

$Ax = 0$

$$\dim(\mathrm{col}(A)) = \dim(\mathrm{row}(A)) = \mathrm{rank}(A)$$

$$\dim(\mathrm{null}(A)) = \mathrm{nullity}(A)$$

$$\underline{\mathrm{rank}}(A) + \mathrm{nullity}(A) = \underline{\#\text{ col of } A}$$

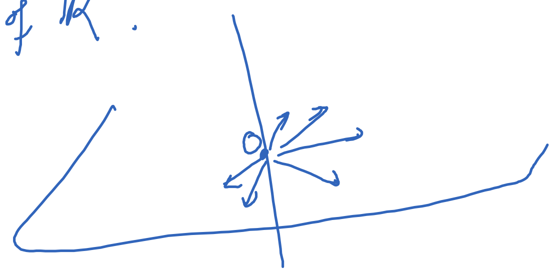
$$17\left\{\underbrace{\begin{bmatrix} \quad & \quad \end{bmatrix}}_{17}\right. \quad 33 \times 67$$

$$A \xrightarrow{RREF} \begin{bmatrix} \quad & \quad \end{bmatrix}$$

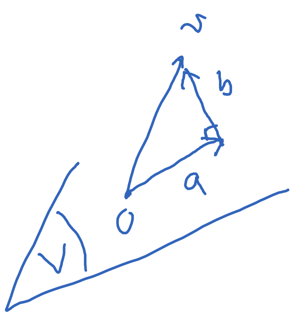
$$\underbrace{V = \text{span}\left\{ \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}2\\0\\1\\3\end{bmatrix} \right\}}_{V^\perp} \quad \text{subspace of } \mathbb{R}^4$$

$$\underbrace{\dim V}_{2} + \underbrace{\dim V^\perp}_{2} = 4$$

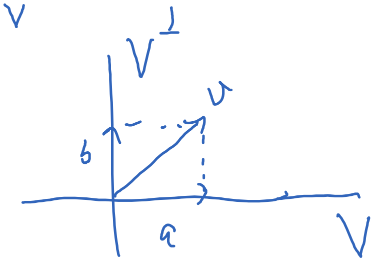
$$V = \text{span}\left\{ \begin{bmatrix}1\\2\\3\end{bmatrix} \right\} \quad \text{subspace of } \mathbb{R}^3.$$



$$\dim V^\perp = 2$$



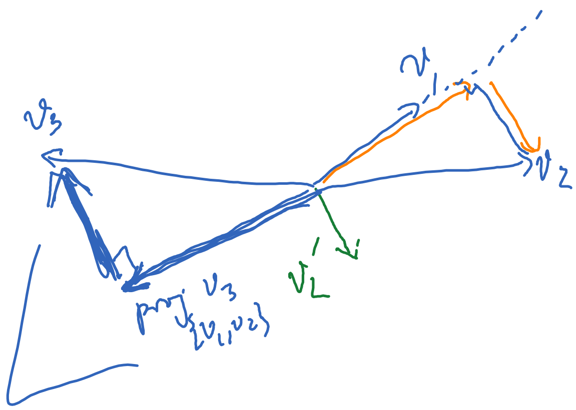
$$v = \underbrace{a}_{\text{proj}_V v} + \underbrace{b}_{\text{perp}_V v = \text{proj}_{V^\perp} v}$$



$$V = \text{span}\{ \ldots\ldots \}$$

$$V = \operatorname{span}\left\{ \begin{array}{c} \overbrace{\hphantom{0}}^{v_1} \\ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \end{array}, \begin{array}{c} \overbrace{\hphantom{0}}^{v_2} \\ \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix} \end{array}, \begin{array}{c} \overbrace{\hphantom{0}}^{v_3} \\ \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \end{array} \right\}$$

Gram-Schmidt process



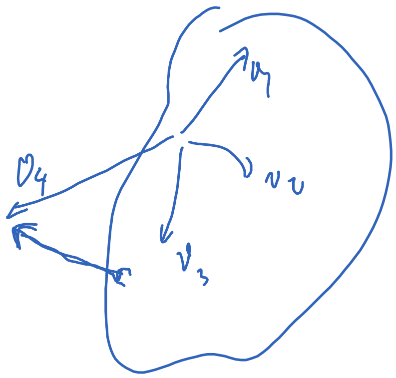
$$v_1 \longrightarrow v_1$$

$$v_2 \longrightarrow v_2 - \underbrace{\operatorname{proj}_{v_1} v_2} = v_2 - \frac{v_1 \cdot v_2}{v_1 \cdot v_1} v_1$$

$$v_3 \longrightarrow v_3 - \underbrace{\operatorname{proj}_{\{v_1, v_2'\}}} v_3 = v_3 - \operatorname{proj}_{v_1} v_3 - \operatorname{proj}_{v_2'} v_3$$

<u>Recall</u>: If $u_1, u_2, \dots, u_n$ are orthogonal to each other, then

$$\operatorname{proj}_{\{u_1, \dots, u_n\}} u = \underbrace{\frac{u \cdot u_1}{u_1 \cdot u_1} u_1}_{\operatorname{proj}_{u_1} u} + \underbrace{\frac{u \cdot u_2}{u_2 \cdot u_2} u_2}_{\operatorname{proj}_{u_2} u} + \dots + \underbrace{\frac{u \cdot u_n}{u_n \cdot u_n} u_n}_{\operatorname{proj}_{u_n} u}$$



$$v_4 \longrightarrow v_4 - \text{proj}_{\{v_1,\, v_2',\, v_3'\}} v_4 = \cdots$$

1. Adding vectors to an orthogonal set to get an orthogonal basis

$$\underbrace{\begin{bmatrix} \vdots \end{bmatrix}, \begin{bmatrix} \vdots \end{bmatrix}}_{2}, \underbrace{\swarrow \ \swarrow \ \swarrow \swarrow}_{48} \qquad 50$$

2. $A \rightsquigarrow$ diagonalize

P, D $\qquad$ QR

A $\qquad$ 17